【論　　文】

システムダンピングを考慮した斜張橋のたわみ風琴振動解析

Characteristic Bending Aeolian Oscillations due to System Damping of Cable-Stayed Bridges

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〔要旨〕　近年，斜張橋の風によって誘起される振動におけるシステムダンピング効果が，我が国においても注目され始めている。著者らは，前に，いわゆる内部共振に起因する現象を支配的な一要因として定義し，走行活荷重による動的応答において，定義した要因によるシステムダンピングが動的増幅率の低減などの効果を及ぼすことを報告している。

本文は，風によって誘起される風のうちでたわみ風琴振動に着目し，著者らが定義した要因によるシステムダンピング効果の検討を目的として，内部共振を考慮した時系列応答解析法を新たに誘導するものである。また，簡単な計算モデルを対象として，その妥当性を検討するものである。

さらに，マルチケーブル型式斜張橋の実橋設計例を対象とし，バネ支持模型風洞実験より得られた非定常空気力係数を用いて，定常状態に至るまでの振幅の発達状況の時系列応答解析を実施するものである。そして，その結果に対する種々の考察から，システムダンピング効果に関する耐風設計上の基礎資料を得ることを試みるものである。

1. Introduction

The cable-stayed girder bridge has been considerably developed in recent years, and the number of this type of structures built in Japan has rapidly increased. For the wind resistant design, the investigation based on the wind-tunnel test is frequently performed in this country.

F. Leonhardt et al.[1),2)] performed the experiments using models and several actual bridges, and pointed out that cable-stayed girder bridges would have high dynamic stability against vertical bending oscillations of main girders by means of the system damping. However, the adduced cause seems to be merely a conjecture. In addition, judging from the results of measurement of many actual bridges built in this country, the system damping effects cannot be considered as characteristics common to all of cable-stayed girder bridges.

On the contrary, the authors[3)-5)] previously defined that a governing cause of the system damping was the beating phenomenon of free vibration terms with respect to two kinds of similarly coupled natural vibration modes (from another angle, a main girder and cables exchanged their vibration energy), when frequencies of a main girder and cables as component structures were close to each other. Namely, it was defined that the system damping would occur when transverse local vibration characteristics of particular cables satisfy the requirements of the so-called internal resonance. Moreover, in dynamic response of cable-stayed girder bridges to moving design live loads, the authors reported that the system damping due to the defined cause would have good effects of reducing the dynamic amplification factors and of giving considerable attenuation of residual free vibrations.

Therefore, it can be positively forecasted that the system damping effects due to the defined cause on wind-induced vibrations may not be ignored, when the aerodynamic stability of cable-stayed girder bridges is discussed. However, in order to examine the effects, it becomes necessary to newly derive an analysis technique of time series response by considering the internal resonance. Because it is difficult to examine, directly by the wind-tunnel test on so-called sectional models, vibrations with respect to two kinds of similarly coupled natural vibration modes which are excited almost at the same time since their natural frequencies are close to each other. Also, in the previous case of dynamic response

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to moving loads, the structural damping was neglected at the safety side and thus the conventional technique could be applied. But, in this case, it become necessary to evaluate the aerodynamic damping as well as the structural damping, differently from the conventional technique, in response to two kinds of coupled modes.

Y. Kubo, M. Ito and T. Miyata[6] performed time series response analyses for flutters by applying the strip theory using unsteady aerodynamic forces obtained from the sectional model test, and reported that the results coincide relatively well with $V–A$ curve (wind velocity – amplitude curve) obtained from the full model test. Therefore, when examining the system damping effects, it is considered to be very valid not only for flutters but also for aeolian oscillations to perform time series response analyses of completed structure models by using unsteady aerodynamic coefficients obtained from the sectional model test.

On the basis of discussions indicated above, now the authors will pay attention to bending aeolian oscillations among various kinds of wind-induced vibrations of cable-stayed girder bridges. And, in this paper, an analysis technique of time series response will be proposed taking into consideration the internal resonance. Then, in order to confirm justification for the proposed technique and the defined cause, action of cable as a kind of damped absorber will be examined by using solution of the complex eigenvalue problem for a simple model simulating cable-stayed girder bridges. In addition, for an actual design example of multi-cable-stayed girder bridges, time series response analyses will be performed using unsteady aerodynamic coefficients obtained from results of the spring-mounted model test, and it will be tried to obtain basic wind resistant design data concerning the system damping effects due to the defined cause.

2. Analysis Technique of Time Series Response by Considering Internal Resonance

In this chapter, an analysis technique of time series response of bending aeolian oscillations on cable-stayed girder bridges will be derived taking into consideration the internal resonance.

This analysis technique is based on the several common assumptions that acting wind has a constant wind velocity and a smooth flow without turbulence, as done in the wind engineering, and that vibration modes during bending aeolian oscillations are satisfied with the assumption of semi-rigid modes, in addition to the following ones:

i) Unsteady aerodynamic forces, obtained from the sectional model test, will be applied but the imaginary part in phase with velocity is predominant and the real part proportional to displacement can be neglected.

ii) Unsteady aerodynamic forces will act in accordance with the strip theory but forces acting on a main girder are predominant and ones acting on cables can be neglected.

At first, to an analytical model of cable-stayed girder bridges with cables replaced by links for considering their transverse local vibrations, the linearlized equation of motion can be expressed by

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=[F_I]\{\dot{y}\} \qquad (1)$$

where $[M]$ is the mass matrix, and $[K]$ is the tangential stiffness matrix in a static equilibrium state. Also, $[C]$ is the damping matrix, and $[F_I]$ is the unsteady aerodynamic damping matrix which will be described later.

The following equation contains no terms proportional to velocity:

$$[M]\{\ddot{y}\}+[K]\{y\}=0\ . \qquad (2)$$

By performing the natural vibration analysis for the above equation, two kinds of the similarly coupled natural vibration modes of i-th and j-th orders, $\{\Phi_i\}$ and $\{\Phi_j\}$, and the natural circular frequencies, ω_i and ω_j which are close to each other, can be obtained due to the internal resonace. If the following normalization has been performed:

$$\{\Phi_k\}^T[M]\{\Phi_k\}=1 \qquad (k=i,j), \qquad (3)$$

Eq.(1) can be transformed into the following 2nd order differential equation relative to the generalized coordinates, q_i and q_j:

$$\ddot{q}_k + 2h_k\omega_k\dot{q}_k + \omega^2{}_k q_k = \{\Phi_k\}^T[F_I]\{\Phi_k\}\dot{q}_k \qquad (k = i, j) \qquad (4)$$

where h_i and h_j are the structural damping constants. But, being different from the conventional cases where only one kind of natural vibration modes is considered, values assumed in the wind-tunnel test cannot be used as these constants. Moreover, these constants must be evaluated by taking account of degree of the internal resonance.

When h_g and h_c, ω_g and ω_c, $[C_g]$ and $[C_c]$, and $\{\Phi_g\}$ and $\{\Phi_c\}$ are respectively the structural damping constants, the natural circular frequencies, the damping matrices and the normalized natural vibration modes of the analytical model of cable-stayed girder bridges neglecting transverse local vibrations of cables and also the model for transverse vibrations of cables, the following equations can be made:

$$2h_k\omega_k = \{\Phi_k\}^T[C_q]\{\Phi_k\}$$

$$= [\{\Phi_{k,g}\}^T, \{\Phi_{k,c}\}^T]\begin{bmatrix}[C_g], & [O]\\ [O], & [C_c]\end{bmatrix}\begin{bmatrix}\{\Phi_{k,g}\}\\ \{\Phi_{k,c}\}\end{bmatrix} \qquad (k = i, j)$$

$$2h_g\omega_g = \{\Phi_g\}^T[C_g]\{\Phi_g\},\ 2h_c\omega_c = \{\Phi_c\}^T[C_c]\{\Phi_c\} \qquad (5)$$

Since the above equations and the fact that $\{\Phi_i\}$ and $\{\Phi_j\}$ are similar to each other, the authors propose here the application of values evaluated by the following approximate expression:

$$2h_k\omega_k = 2h_g\omega_g\,[\{\Phi_{k,g}\}^T\{\Phi_{k,g}\}]\,/\,[\{\Phi_g\}^T\{\Phi_g\}]$$

$$+ 2h_c\omega_c\,[\{\Phi_{k,c}\}^T\{\Phi_{k,c}\}]\,/\,[\{\Phi_c\}^T\{\Phi_c\}] \qquad (k=i, j) \qquad (6)$$

where $\{\Phi_{k,c}\}$ expresses only the transverse local vibration component of the cables in $\{\Phi_k\}$, and $\{\Phi_{k,g}\}$ expresses $\{\Phi_k\}$ from which this component has been deducted. Justification for evaluating the structural damping constants by using Eq.(6) will be confirmed in the next chapter.

Hence, by giving the unsteady aerodynamic damping matrix $[F_I]$, Eq.(4) can be expressed definitively. This matrix can be calculated by using the unsteady aerodynamic lift coefficient C_{LZI} obtained from results of the sectional model test with the dimentionless amplitude Z_r and the reduced wind velocity U_r at each angle of attack. However, differently from the conventional cases, it is required to give U_r and Z_r corresponding to two kinds of the natural circular frequencies close to each other and the similarly coupled natural vibration modes. These values may be computed as follows. Namely, the reduced wind velocity U_r may be given corresponding to the average value $(\omega_i+\omega_j)/2$. On the other hand, the dimensionless amplitude Z_r may be given corresponding to the equivalent amplitude vector $\{Z_p\}$ which can be expressed by the equation shown below in each time step.

$$\{Z_p\} = \sqrt{[q_i\{\Phi_i\} + q_j\{\Phi_j\}]^2 + \left[\frac{\dot{q}_i\{\Phi_i\} + \dot{q}_j\{\Phi_j\}}{(\omega_i + \omega_j)/2}\right]^2} \qquad (7)$$

Therefore, by integrating Eq.(4) in succession with small interval and applying the mode superposition method, time series response analyses of bending aeolian oscillations by considering the internal resonance can be performed.

3. Justification of Analysis Technique and Cable Action as Damped Absorber

In this Chapter, in order to confirm justification for evaluating the structural damping constants by using Eq. (6), cable action as a kind of damped absorber[7] will be examined by using solution of the complex eigenvalue problem concerning a simple simulation model of cable-stayed girder bridges. Moreover, the total damping constant to each mode, including the aerodynamic damping, will be concretely evaluated, and compared with the one in the ordinary case.

3–1 Justification of Analysis Technique

By substituting reasonable values of h_g and h_c into Eq. (6), the following fact will be known easily. Namely, it will be known that the sum of h_i and h_j corresponding to the coupled natural vibration modes due to the internal resonance is equal to the sum of h_g and h_c. This fact apparently means that the structural damping is added to the vibration of the main girder shaped by the two kinds of the similarly coupled natural vibration modes which are excited almost at the same time by forces acting on the main girder, when the remarkable internal resonance occurs. And, by considering that a cable will act as a kind of damped absorber shown in Fig.1, this fact can be estimated as explained below.

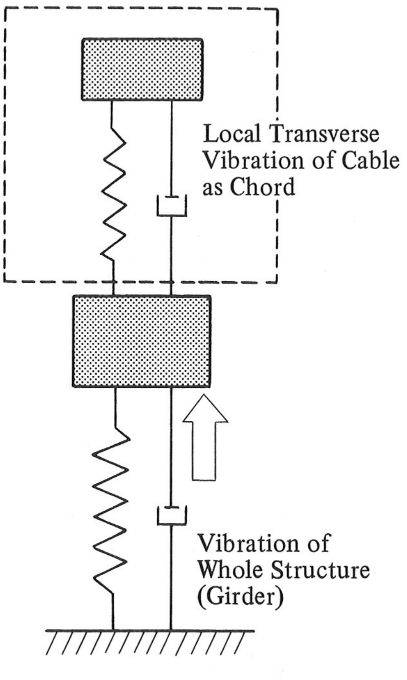


Fig.1 Damped Absorber

Fig.2 shows the simple simulation model of cable-stayed girder bridges being considered. For the purpose of simplification, the horizontal displacement of the free end of the main girder is ignored, and then the vibratory system with 2-degree of freedom relative to the displacement x_g in the vertical direction and the one x_c in the normal direction at the central point of the cable is considered. In this figure, c_g and c_c are the structural damping coefficients, and m_g and m_c are the mass of the main girder and the cable respectively.

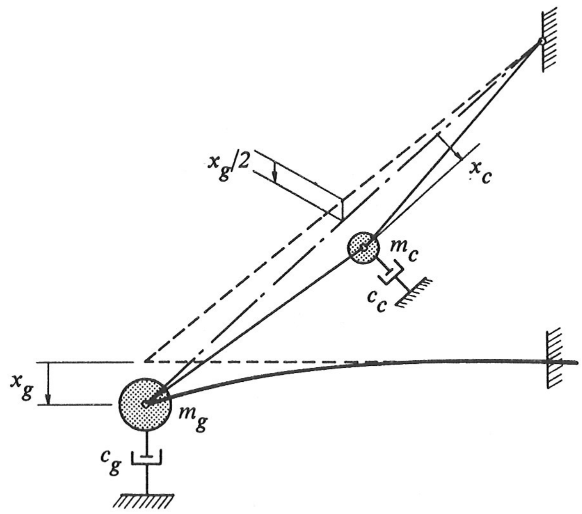


Fig.2 Simulation Model of Cable-Stayed Beams

The equation of motion of the free damping vibration may be given by the following equation:

$$\begin{bmatrix} m_g+m_c/2 & 0 \\ 0 & m_c \end{bmatrix}\begin{bmatrix} \ddot{x}_g \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} c_g & 0 \\ 0 & c_c \end{bmatrix}\begin{bmatrix} \dot{x}_g \\ \dot{x}_c \end{bmatrix} + \begin{bmatrix} k_{gg} & k_{gc} \\ k_{cg} & k_{cc} \end{bmatrix}\begin{bmatrix} x_g \\ x_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{8}$$

where k_{gg} and k_{cc}, and k_{gc} and k_{cg} express the diagonal elements and the non-diagonal ones of the stiffness matrix respectively, but actual expressions for these elements are omitted here.

If the model as shown in Fig.3, in which the transverse vibration of the cable is neglected, is presumed here, the equation of motion shown below is given from Eq. (8).

$$(m_g + m_c/2)\, \ddot{x}_g + c_g \dot{x}_g + k_{gg} x_g = 0 \tag{9}$$

Then, when ω_g and h_g are respectively the natural circular frequency of the model of 1-degree of freedom system and the structural damping constant, the following equation is obtained:

$$\left.\begin{aligned} c_g &= 2(m_g + m_c/2)h_g\omega_g \\ k_{gg} &= (m_g + m_c/2)\omega_g^{\,2} \end{aligned}\right\} \qquad (10)$$

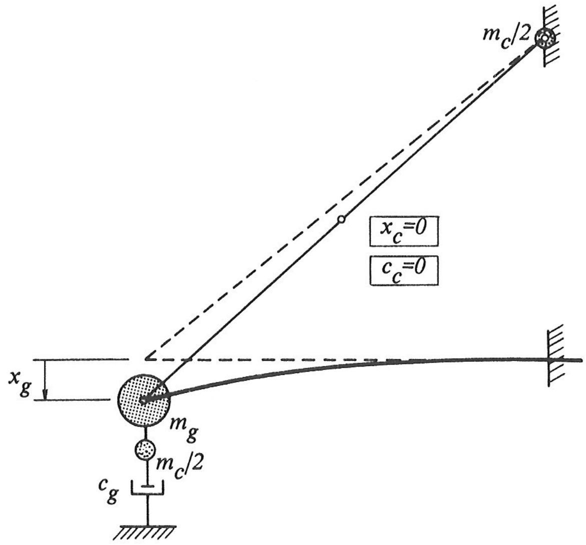


Fig.3 Model in which Transverse Vibration of Cable is Neglected

On the other hand, if the model with only the transverse vibration of the cable as shown in Fig.4 is presumed, the equation of motion show below is given from Eq. (8) in the same way.

$$m_c\ddot{x}_c + c_c\dot{x}_c + k_{cc}x_c = 0 \qquad (11)$$

Then, when ω_c and h_c are respectively the natural circular frequency of this model of 1-degree of freedom system and the structural damping constant, the following equation is obtained:

$$\left.\begin{aligned} c_c &= 2m_ch_c\omega_c \\ k_{cc} &= m_c\omega_c^{\,2} \end{aligned}\right\} \qquad (12)$$

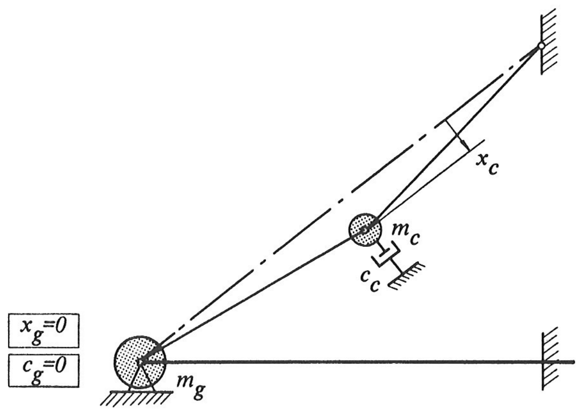


Fig.4 Model with Only Transverse Local Vibration of Cable

As an extreme case where the internal resonance occurs, the following assumption is made here:

$$\omega_g = \omega_c = \overline{\omega} \qquad (13)$$

Then, by substituting Eqs. (10), (12), (13) and the equation shown below as the conventional approach into Eq. (8)

$$\left.\begin{aligned} x_g &= X_g e^{i\Omega t} \\ x_c &= X_c e^{i\Omega t} \end{aligned}\right\}, \qquad (14)$$

the following equation is obtained:

$$e^{i\Omega t}\begin{bmatrix} -\Omega^2+2ih_g\overline{\omega}\Omega+\overline{\omega}^2 & k_{gc}/(m_g+m_c/2) \\ k_{cg}/m_c & -\Omega^2+2ih_c\overline{\omega}\Omega+\overline{\omega}^2 \end{bmatrix}\begin{bmatrix} x_g \\ x_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (15)$$

From the condition of having significant solutions of the above equation with respect to an unknown quantity Ω can be derived as follows:

$$\Omega^4 - 2i(h_g+h_c)\overline{\omega}\Omega^3 - \{2+(h_g+h_c)^2 - \underline{(h_g-h_c)^2}\}\overline{\omega}^2\Omega^2$$
$$+2i(h_g+h_c)\overline{\omega}^3\Omega + \overline{\omega}^4 - \alpha^2 = 0 \qquad (16)$$

Where,

$$\alpha = \{k_{gc} / (m_g + m_c/2)\} (k_{cg}/m_c). \quad (17)$$

Also, by using the following condition:

$$\left.\begin{array}{l} 0 < h_g \ll 1.0 \\ 0 < h_c \ll 1.0 \end{array}\right\}, \quad (18)$$

Eq. (16) can be transformed into the equation shown below after neglecting its underlined small term.

$$\{\Omega^2 - 2i(\frac{h_g+h_c}{2})\bar{\omega}\Omega - (\bar{\omega}^2 - \alpha)\}\{\Omega^2 - 2i(\frac{h_g+h_c}{2})\bar{\omega}\Omega - (\bar{\omega}^2 + \alpha)\} = 0 \quad (19)$$

Hence, two sets of the conjugate complex roots Ω_1 and Ω_2 can be obtained as follows:

$$\left.\begin{array}{l} \Omega_1 \fallingdotseq i(\frac{h_g+h_c}{2})\bar{\omega} \pm \sqrt{\bar{\omega}^2\{1-(\frac{h_g+h_c}{2})^2\} - \alpha} \\ \Omega_2 \fallingdotseq i(\frac{h_g+h_c}{2})\bar{\omega} \pm \sqrt{\bar{\omega}^2\{1-(\frac{h_g+h_c}{2})^2\} + \alpha} \end{array}\right\} \quad (20)$$

Moreover, the coupling ratios $(X_c/X_g)_1$ and $(X_c/X_g)_2$ in the vibration mode corresponding to each of Ω_1 and Ω_2 can be obtained from Eq. (15) as follows:

$$(X_c/X_g)_1 = -(X_c/X_g)_2 = \frac{\alpha}{k_{gc}/(m_g+m_c/2)} = \sqrt{\frac{(m_g+m_c/2)/k_{gc}}{m_c/k_{cg}}} \quad (21)$$

From Eq.(20), it will be known that the sum of the imaginary parts of Ω_1 and Ω_2 becomes $i(h_g + h_c)\bar{\omega}$. This means the confirmation of the fact that cable action as a kind of damped absorber gives the effects of varying the structural damping to the vibration of the main girder shaped by the two kinds of the similarly coupled natural vibration modes, which have the natural circular frequencies $\omega_1 = \sqrt{\bar{\omega}^2 - \alpha}$ and $\omega_2 = \sqrt{\bar{\omega}^2 + \alpha}$ close to each other and are excited almost at the same time by forces acting on the main girder. Namely, this confirmation can be easily done because the conjugate complex root Ω_g is given by the following equation when the transverse vibration of the cable is neglected:

$$\Omega_g = ih_g\bar{\omega} \pm \sqrt{\bar{\omega}^2(1 - h_g^2)} \quad (22)$$

While, from Eq. (21), it will be known that the coupling ratio X_c/X_g is depending on the mass ratio m_c/m_g and thus the mass ratio greater than a predetermined value will become necessary.

Therefore, it can be judged from the above that justification for evaluating the structural damping constants by Eq. (6) has been confirmed, as one of the features of the time series response analysis technique for bending aeolian oscillations by considering the internal resonance.

3–2 Total Damping Constant Including Aerodynamic Damping

From Eq.(21), the normalized natural vibration modes, $\{\Phi_1\}$ and $\{\Phi_2\}$, can be obtained as follows:

$$\left.\begin{array}{l} \{\Phi_1\} = [\sqrt{1/\{2(m_g+m_c/2)\}}, -\sqrt{1/(2m_c)}]^T \\ \{\Phi_2\} = [\sqrt{1/\{2(m_g+m_c/2)\}}, +\sqrt{1/(2m_c)}]^T \end{array}\right\} \quad (23)$$

Hence, by paying attention to bending aeolian oscillations due to wind forces acting on the main girder, the following equation may be expressed for each time step:

$$\{\Phi_1\}^T[F_I]\{\Phi_1\} = \{\Phi_2\}^T[F_I]\{\Phi_2\} = 1/2(\{\Phi_g\}^T[F_I]\{\Phi_g\}) \quad (24)$$

where $[F_I]$ is the unsteady aerodynamic damping matrix which was described in the above chapter, and $\{\Phi_g\} = [\sqrt{1/(m_g + m_c/2)},\ 0]^T$ is the normalized natural vibration mode when the transverse vibration of the cable is neglected.

Therefore, it will be known that the following condition can be easily satisfied, and that the total damping constant to each mode is increased as compared with the case where the internal resonance hardly occurs:

$$\left.\begin{aligned} 2\{(h_g+h_c)/2\}\omega_1-h_g{}^*/2 &> 2h_g\omega_g-h_g{}^* \\ 2\{(h_g+h_c)/2\}\omega_2-h_g{}^*/2 &> 2h_g\omega_g-h_g{}^* \end{aligned}\right\} \qquad (25)$$

where $h_g{}^*=\{\Phi_g\}^T[F_I]\{\Phi_g\}$. Consequently, it can be judged that the defined cause will be one of the governing factors of the system damping of cable-stayed girder bridge.

4. Time Series Response Analyses of Multi-Cable-Stayed Girder Bridges

In this chapter, time series response analyses with respect to an actual design example will be performed using unsteady aerodynamic coefficients obtained from the spring-mounted model test, and then it will be tried to obtain basic data for the wind resistant design related to the system damping effects on bending aeolian oscillations of cable-stayed girder bridges.

Also, for the purpose of comparison, calculations will be performed by changing characteristics of transverse vibrations of particular cables. However, since the adjustment of cable tension is not always easy because of the connection with the static design, the increment of unit weight per length of cable, that is, the additional mass will be considered here to correspond to actual problems.

4–1 Actual Example and Analysis Procedure

An actual design example to be considered is a 3-span multi-cable-stayed girder bridge[8] with a center span of 420 m, and its skeleton and sectional values are as shown in Fig.5 and Table 1. However, the dimensions for the main girder were obtained after converting truss sections into beam sections.

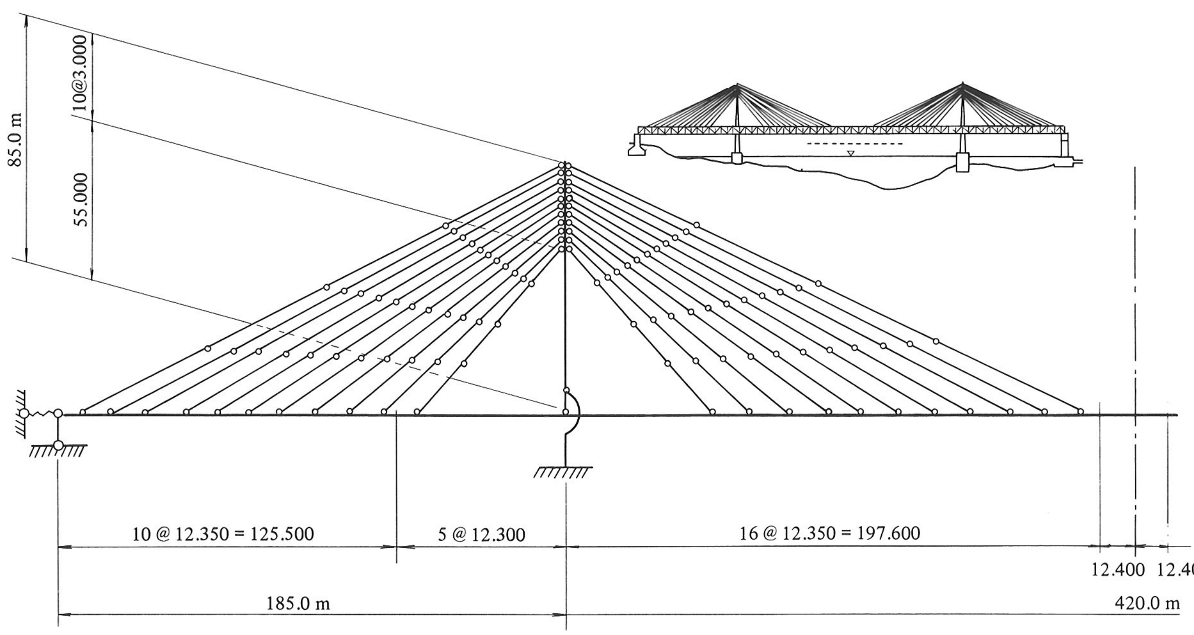


Fig.5 Multi-Cable-Stayed Girder Bridge

Table 1 Sectional Values

	AREA (m^2)	INERTIA (m^4)	Y.MODULUS (t/m^2)
GIRDER	1.0635–1.5747	40.538–65.585	21000000.0
TOWER	1.4780–2.2520	4.568– 9.054	21000000.0
CABLE	0.0225–0.0419	0.0	20500000.0
SPRING (length:3.0m)	1.0	0.0	36000.0

In the natural vibration analysis, cable replaced by links with required non-stressed shapes will be considered, and the equation of motion linearized by the tangential stiffness matrix[10] based on the finite displacement theory under static dead load will be solved by Sturm Sequence Method[11].

Also, in the time series response analysis, the symmetric 1st order vibration mode of the main girder will be specifically taken into account, and the analysis will be made by the mode superposition method applying Newmark's β-Method ($\beta=1/6$)[12] as numerical integration method. In this case, the small duration of time step will be 1/20 of the natural vibration period being considered and, if the two kinds of the similary coupled natural vibrations modes are to be considered by taking account of the internal resonance, then 1/20 of the average period will be used.

4–2 Calculation Model and Natural Vibration Characteristics

Table 2 shows the natural vibration period when all cables at all levels are considered to be the tension members and the transverse vibration is neglected. On the other hand, Table 3 shows characteristics of transverse vibration of cables as chords at all levels.

Table 2 Natural Periods by Neglecting Transverse Vibrations of Cables

ORDER	PERIOD (sec)	MODE
1st	2.437	Center Span – Symmetric 1st (vertical)
2nd	2.406	Center Span – Symmetric 1st (longitu.)
3rd	1.375	Center Span – Asymmetric 1st (vertical)
4th	0.910	Side Span – Symmetric 1st (vertical)
5th	0.738	Side Span – Asymmetric 1st (vertical)

Table 3 Transverse Local Vibration Characteristics of Cables

		AREA (m^2)	DENSITY (t/m^3)	TENSION (t)	PERIOD (sec)
CENTER SPAN	11th	0.03586	11.0	1507.	1.953
	10 Upper	0.03586	11.0	1490.	1.866
	9	0.03186	11.0	1188.	1.655
	8	0.03186	11.0	981.	1.536
	7	0.02848	11.0	977.	1.481
	6	0.02248	11.0	774.	1.355
	5	0.02248	11.0	770.	1.228
	4	0.02248	11.0	760.	1.106
	3	0.02248	11.0	754.	0.984
	2 Lower	0.02848	11.0	1124.	0.887
	1st	0.02848	11.0	1243.	0.763
SIDE SPAN	11th	0.04186	11.0	1544.	2.132
	10 Upper	0.04186	11.0	1531.	2.003
	9	0.03972	11.0	1120.	1.846
	8	0.03186	11.0	1112.	1.590
	7	0.02248	11.0	807.	1.431
	6	0.02248	11.0	800.	1.312
	5	0.02248	11.0	795.	1.192
	4	0.02248	11.0	784.	1.076
	3	0.02248	11.0	775.	0.691
	2 Lower	0.02848	11.0	1151.	0.881
	1st	0.02848	11.0	1267.	0.762

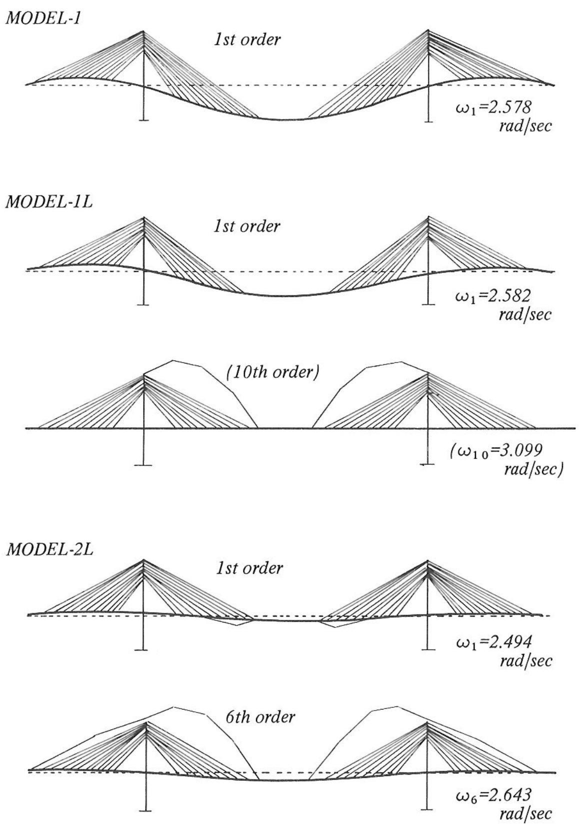
By referring to these tables, four kinds of calculation models shown in Table 4 will be used here. That is, the specifications of actual design examples of bridges will be used as they are for *MODEL-1* and *MODEL-1L*, which correspond to the transverse vibration being neglected and considered. *MODEL-2L* corresponds to the case where the requirements for the internal resonance are satisfied by adding the mass to the four uppermost cables at 11th level and making 1st order natural vibration period of the cables as chords very close to that of the lowest order shown in Table 2. And *MODEL-3L* corresponds to the case where the same mass is added also to four cables at 10th level in addition to the 11th level.

Table 4 Calculation Models

	DENSITY OF CABLE (t/m^3)				
	1st–9th	10th		11th	
		CENTER SPAN	SIDE SPAN	CENTER SPAN	SIDE SPAN
MODEL-1 (AXIAL MEMBER)	11.000	11.000		11.000	
MODEL-1L (LINKING CABLE)	11.000	11.000		11.000	
MODEL-2L (LINKING CABLE)	11.000	11.000		16.192	14.025
MODEL-3L (LINKING CABLE)	11.000	18.282	15.873	16.192	14.025

Increment of the unit weight per length of cable shown in Table 4, that is, the ratio of added mass to the standard value is 1.275 to 1.662. And it seems to be actually possible to satisfy the requirements for internal resonance by adjusting the thickness of grouting in the case of HiAm-anchor cables or by adjusting the weight of band for binding strands in the case of other cables.

Therefore, the natural vibration modes and natural circular frequencies concerning the 1st order symmetric vibration of the main girder are shown in Fig.6 with respect to these four kinds of calculation models. That is, the mode superposition method will be applied by paying attention to the natural vibration modes shown in this figure. However, in the case of *MODEL-1L*, the internal resonance hardly occurs even if the transverse vibration of the cables is taken into accout. Natural vibration mode in the 10th order shown in parenthesis will not be excited by the unsteady aerodynamic force acting on the main girder but is indicated there as reference.



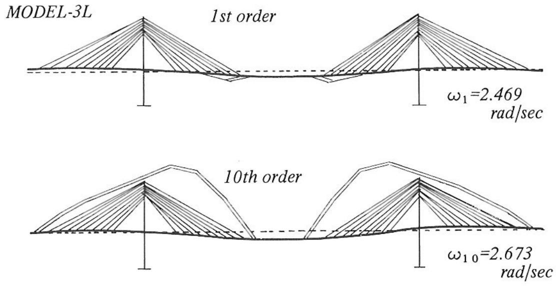


Fig.6 Natural Vibration Modes and Natural Circular Frequencies

4–3 Structural Damping Constants and Unsteady Aerodynamic Forces

By considering four cases shown in Table 5, the values of structural damping constants h_i and h_j corresponding to each mode in the time series response analysis were calculated. That is, for *CASE-1, -2, -3* and *-4*, a value of 0.03 was used as the structural logarithmic decrement δ_g corresponding to the models for which the transverse vibration of cables was neglected, and the value of δ_c corresponding to the models with only the transverse vibration of cables was changed from 0.0 to 0.0075, to 0.0150 and to 0.0300.

Table 5 Structural Damping Constants Corresponding to Each Mode

CASE	ASSUMED VALUE OF LOGARITHMIC DECREMENT		MODEL	DAMPING CONSTANT TO EACH MODE				
	$\delta_g=2\pi h_g$	$\delta_c=2\pi h_c$		i-th	h_i	j-th	h_j	Total
CASE-1	0.03	0.0	*MODEL-1L*	1st	0.00477	–	–	0.00477
			MODEL-3L	1st	0.00173	10th	0.00304	0.00477
CASE-2	0.03	0.0075	*MODEL-1L*	1st	0.00477	–	–	0.00477
			MODEL-3L	1st	0.00245	10th	0.00352	0.00597
CASE-3	0.03	0.0150	*MODEL-1L*	1s.	0.00477	–	–	0.00477
			MODEL-3L	1st	0.00317	10th	0.00399	0.00716
CASE-4	0.03	0.0300	*MODEL-1*	1st	0.00477	–	–	0.00477
			MODEL-1L	1st	0.00477	–	–	0.00477
			MODEL-2L	1st	0.00499	6th	0.00456	0.00955
			MODEL-3L	1st	0.00458	10th	0.00497	0.00955

On the other hand, as the unsteady aerodynamic lift coefficient, C_{LZI}, a value corresponding to the reduced wind velocity U_r=1.992 shown in Fig.7 is applied, which was formulated by the least square method by using $V\text{–}A\text{–}\delta$ curve (wind velocity-amplitude - logarithmic decrement curve)[9] obtained from the results of wind tunnel test on the spring-mounted model with truss girder in the case of an angle of attack of 5-degrees. Ratio of C_{LZI} to dimensionless amplitude Z_r is also shown in Fig.7 for reference.

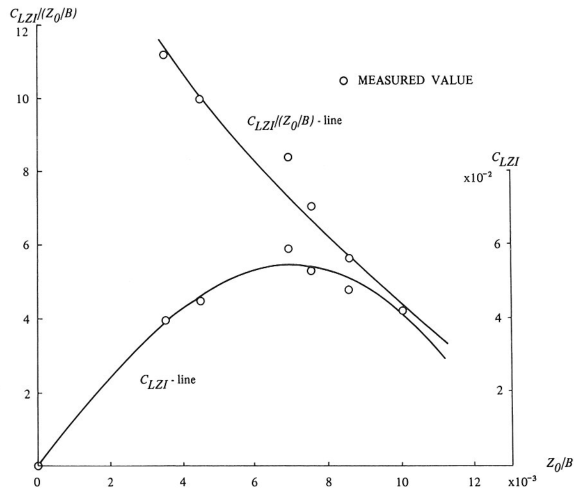


Fig.7 Unsteady Aerodynamic Coefficient

4–4 Calculated Results and Its Consideration

Part of calculated results of the time series response analysis is shown in Fig.8. Fig.8 shows the envelope of the response amplitude from the initial development stage to steady state of the vertical displacement at ½ point of uppermost cable at ½ point of center span of the main girder, when the initial value of vertical displacement amplitude at ½ point of the center span of the main girder is 0.150 m and periodicity is 100 (number of time steps is 2000).

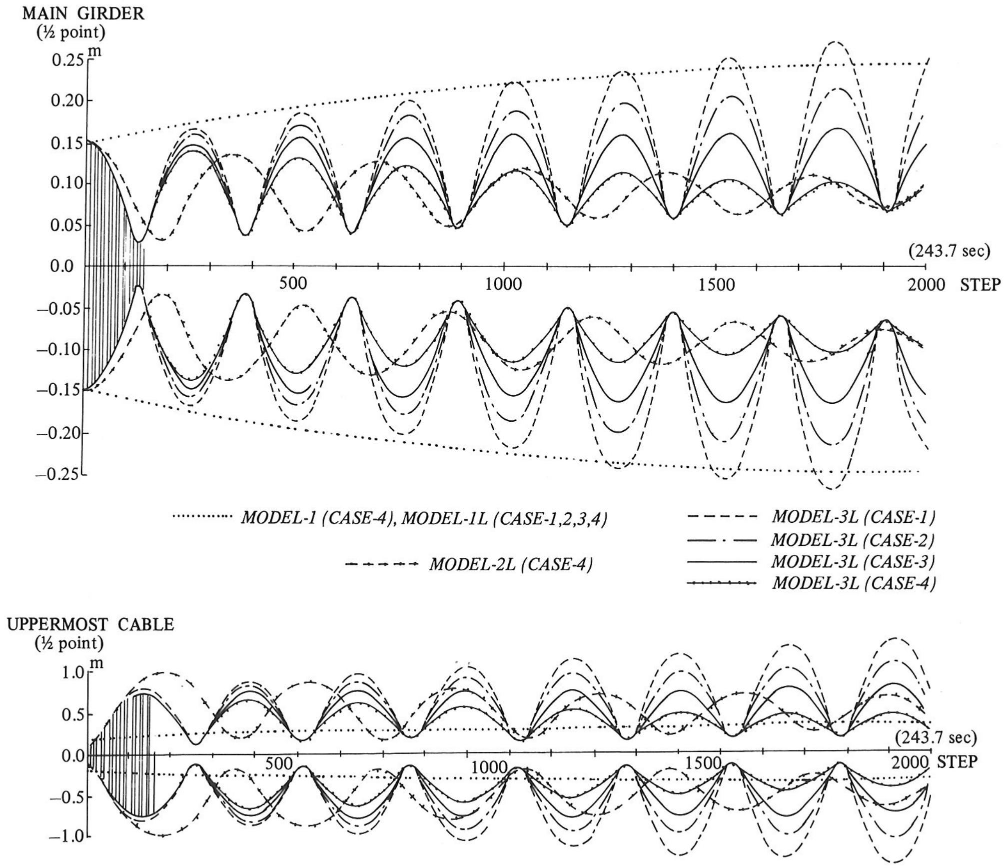


Fig.8 Increase of Response Amplitude of Girder or Cable

From this figure, the following consideration can be made: There is almost no difference in response amplitude of bending aeolian oscillations of main girder between *MODEL-1* and *MODEL-1L*. On the other hand, it will be known that, in the case of *MODEL-2L* and *-3L*, the response amplitude is gradually decreased periodically. And in the case of *MODEL-3L*, the response amplitude of vibration of the cable becomes smaller compared to *MODEL-2L*.

In the case of *MODEL-3L*, it will be known that the steady-state response amplitude slightly increases in *CASE-1* where cable action as a kind of damped absorber is neglected but becomes considerably small in *CASE-2* and *-3* compared to *MODEL-1L*. It will be also known that, in *CASE-4*, the development of aeolian oscillations is restricted in both *MODEL-2L* and *-3L*. And in *MODEL-3L*, the response amplitude of vibration of cable decreases as the cable action as a kind of damped absorber is enhanced from *CASE-1* to *CASE-4*.

Therefore, from the results of analysis and its consideration explained above, it seems to be very worthwhile in the wind resistant design to consider the system damping effects on the bending aeolian oscillations of the cable-stayed girder bridges which have been definitively pointed out by the analysis technique derived.

5. Conclusions

From the afore-mentioned results, the following conclusions[13), 14)] may be drawn:

(1) It is positively predictable that the review of the system damping effects due to the defined cause will become not negligible in the wind resistant design of cable-stayed girder bridges. And, for this review, the proposed analysis technique of time series response of bending aeolian oscillations is effective, which takes account of the internal resonance and uses unsteady aerodynamic coefficients obtained from results of the wind-tunnel test on sectional models.

(2) By satisfying the requirements for the internal resonance by adjusting distributed mass of particular cables of multi-cable-stayed girder bridges, the system damping effects are able to considerably reduce the steady state amplitude in bending aeolian oscillations of main girders and are effective for improving the aerodynamic stability.

(3) The system damping effects can be also effectively utilized when it becomes necessary to review the fatigue strength of cable-stayed girder bridges concerning bending aeolian oscillations, because the steady state amplitude of main girders is slowly and repeatedly decreased at the period of the beating phenomenon.

(4) By enhancing the action of cables as a kind of damped absorber by using cables with higher damping capacities, it is not difficult to even restrict the occurrence of bending aeolian oscillations of main girders of cable-stayed girder bridges by means of the system damping effects.

Though the attention is concentrated only upon bending aeolian oscillations among various kinds of wind-induced vibrations in this paper, it seems to be necessary to review the system damping effects peculiar to cable-stayed girder bridges on torsional aeolian oscillations and also on flutters as future research themes.

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